

The World as a Dual Josephson Junction

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Abstract

We examine some of the implications of the field-theoretical mechanism for the localization of gauge fields on hypersurfaces in higher-dimensional bulk space-time. This mechanism exploits the analogy between confinement and dual superconductivity. In the simplest case of a photon localized on a (2+1)-dimensional surface in a (3+1)-dimensional bulk, we argue that the system behaves like a dual Josephson junction. This implies that the effective gauge theory on the surface is not free, but displays weak confinement with a linear potential. We comment on the relevance of our results for the realistic case of a (3+1)-dimensional surface in a space-time with one or more extra dimensions.

The possibility that the particles of the Standard Model are localized on a (3+1)-dimensional hypersurface in a higher-dimensional bulk space-time has been considered repeatedly in the past. The localization of fermions [1] relies on an index theorem [2] that guarantees the presence of fermionic zero modes at the points where a background scalar field, with a Yukawa coupling to the fermions, vanishes. This mechanism can be employed in order to simulate chiral fermions on the lattice by attaching them to a (3+1)-domain wall in the (4+1)-dimensional bulk [3]. The efficiency of the mechanism has been verified through lattice simulations [4].

The localization of gauge fields, which will be our subject of interest, is more difficult to achieve in a field-theoretical context. The most promising proposal [5]–[7] suggests that the low-energy gauge fields are trapped at the center of a defect by being massless there, while they become very massive in the bulk. In order to guarantee that the localized fields are long-ranged within the defect, the effective mass must be generated by embedding them in the gauge sector of a theory that is confining in the bulk. This mechanism has been employed in the recent investigations of the possible presence of extra dimensions experimentally accessible at TeV energies [7, 8]. Other aspects have been explored in refs. [9].

In this work we examine in detail some of the implications of the gauge-field localization mechanism. Our discussion is limited to the field-theoretical scenario¹. We consider the simplest case of a $U(1)$ gauge symmetry. Similarly to refs. [6, 7], we discuss explicitly the reduction of a (3+1)-dimensional theory to a lower-dimensional one. We are forced to do so by the nature of the reduction mechanism, which involves strongly coupled regimes of gauge theories. As only a limited understanding of these regimes is available in 3+1 dimensions, with no concrete generalization to higher dimensions, we limit ourselves to the most accessible case. The implications for the reduction of a higher-dimensional theory to a (3+1)-dimensional one can be inferred only by analogy.

Firstly we review how the photon of the $U(1)$ gauge symmetry is localized on the brane. We then argue that, in analogy with the behaviour in standard Josephson junctions, the resulting (2+1)-dimensional theory is not free. The equation of motion for the electromagnetic field includes a small effective mass term that indicates the presence of a finite correlation length. Moreover, configurations exist that resemble lines of electric flux. The most consistent interpretation of this behaviour is that the system displays weak confinement with a linear potential.

The situation of interest is schematically presented in fig. 1. Within a (3+1)-dimensional bulk space-time we consider a brane² of finite thickness d along the z -axis. We assume that the gauge field remains massless at tree level within this region, while it develops a large effective mass in the bulk. At energy scales below this mass we expect an effective (2+1)-dimensional theory to appear on the brane. A first guess would be that this theory is a free $U(1)$ gauge theory with a massless photon. However, the nature of the reduced theory depends on the mechanism through which the gauge field develops an effective mass in the bulk.

An implementation of the above scenario [5] is obtained in the context of the Abelian Higgs model, described by the tree-level Lagrangian

$$\mathcal{L}_{tr} = \int d^4x \left\{ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + [(\partial_\mu + iqA_\mu)\phi]^* [(\partial^\mu + iqA^\mu)\phi] - V(\phi) \right\}, \quad (1)$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We work in Minkowski space with metric $(+, -, -, -)$. In a simple configuration that could localize the gauge field, the scalar field ϕ has a zero expectation value

¹ Within string theory, matter and gauge fields can be localized on D-branes [10], with the emergence of low-energy (3+1)-theories, weakly coupled to the modes propagating in the extra dimensions [11]. The possibility of experimentally accessible extra dimensions can be considered within this framework as well [12].

² We borrow string-inspired terminology, even though our discussion is limited to the field-theoretical scenario.

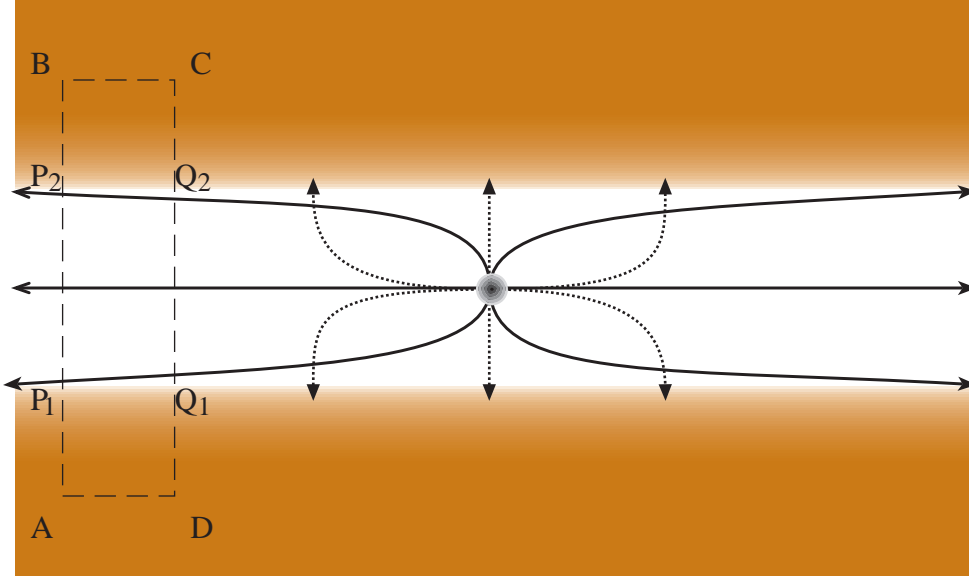


Figure 1: *Electric (dotted) and magnetic (solid) field lines generated by a fictitious dyon (electrically and magnetically charged particle) in the Josephson junction. In the dual picture the roles of electric and magnetic fields are reversed. The x -axis is horizontal and the z -axis vertical.*

inside the brane and a constant non-zero value in the bulk. It is not important how such a configuration is generated dynamically. It may result from the interaction of ϕ with other fields [5]. For our discussion we shall assume that $\phi = 0$ for $|z| < d/2$ and $\phi = \rho \exp(i\theta)$ for $|z| \geq d/2$. As a result, the gauge field becomes massive in the bulk through the Higgs mechanism, while it remains massless inside the brane.

The Abelian Higgs model in the spontaneously broken phase is equivalent to the Ginzburg-Landau theory of superconductivity. A well known property of superconductors is the existence of frictionless currents that can flow without a potential. For the tree-level Lagrangian of eq. (1) the current density is given by

$$J_{tr}^\mu = -2q\rho^2 \partial^\mu \theta - 2q^2 \rho^2 A^\mu. \quad (2)$$

The phase θ is unobservable for a bulk superconductor, as it can be eliminated through a gauge transformation. The superconducting currents flow near the surface of the material and expel magnetic fields from it. This property, the Meissner effect, can be understood also as a consequence of the photon mass $\sqrt{2}q\rho$. The magnetic field decays over a distance $\lambda \sim (q\rho)^{-1}$. Deep inside the superconductor, the gauge field is zero and the current of eq. (2) vanishes.

The configuration of fig. 1 corresponds to a Josephson junction: two superconducting regions separated by a thin layer of non-superconducting material. The phase θ cannot be eliminated completely in this case. More specifically, we can define the gauge-independent phase difference

between two points [13]

$$\Delta\theta_{P_1 P_2} = \theta(P_2) - \theta(P_1) - q \int_{P_1}^{P_2} \vec{A} \cdot d\vec{l}. \quad (3)$$

The gauge-dependent phases $\theta(P_1)$ and $\theta(P_2)$ can be eliminated for convenience through an appropriate gauge transformation. The phase difference $\Delta\theta_{P_1 P_2}$ between two points on either side of the junction can be non-zero [13]. Of particular interest to us will be the phase difference generated by the existence of an electromagnetic field in the non-superconducting region (see below).

As a result of the presence of a phase difference, a superconducting current flows across a junction, the Josephson current. This can be shown on general grounds [14] by noticing that, beyond tree level, the effective action of the system depends on $\Delta\theta$ through quantum effects (tunnelling of charges across the junction). The current can be obtained from the effective matter action through differentiation with respect to the gauge field. The dependence of the action on a phase difference given by eq. (3) immediately leads to the presence of a current. This tunnelling current does not require a potential difference between the two superconducting regions in order to flow.

We can change each of the phases $\theta(P_1)$, $\theta(P_2)$ by a multiple of 2π without altering its physical significance. This means that the effective action and, therefore, the Josephson current must be a periodic function of $\Delta\theta$. We parametrize the current density as [13, 14]

$$J^3(\Delta\theta) = J_{\max}^3 \sin(\Delta\theta), \quad (4)$$

where J_{\max}^3 is its maximum value. We emphasize that J^3 is a tunnelling current. (The classical current of eq. (2) is zero in the brane.) This makes its calculation very difficult, as it depends on the form of the barrier that separates the two bulk regions with non-zero condensates. However, eq. (4) is sufficient for our discussion. The maximum value J_{\max}^3 can be taken as a phenomenological parameter that can be very small in units of the typical mass scale of the potential $V(\phi)$ in eq. (1).

We can now consider the behaviour of the electromagnetic field inside the brane. We employ the Maxwell equations in the presence of an external current density given by eq. (4). They correspond to the equations of motion derived from the tree-level Lagrangian of eq. (1) with $\phi = 0$ and an external current. The presence of the bulk superconductors imposes certain conditions on the solutions of these equations. The electric field parallel to a conductor is zero near its surface. As pointed out in ref. [5, 6], this means that electric field lines must end perpendicularly to the boundary of the Josephson junction. For a point charge, the electric field (whose lines are denoted by the dotted lines in fig. 1) dies off within a distance $\sim d$ in the x, y -directions. As we are interested in the low-energy behaviour of the system, we do not consider configurations with variations of the fields at short distances. This means that we can approximate E_x , E_y as zero and assume that E_z is independent of z inside the brane. The magnetic field has a continuous z -component at the surface of a conductor. As it is zero inside the superconductor, we assume that B_z vanishes everywhere. The components B_x , B_y are non-zero in the brane and vanish exponentially within a distance $\lambda \sim (q\rho)^{-1}$ in the bulk. The magnetic field lines are localized inside the brane (solid lines in fig. 1).

In terms of the gauge field A^μ the above conditions can be written as

$$E_x = -\frac{\partial A^0}{\partial x} - \frac{\partial A^1}{\partial t} = 0 \quad E_y = -\frac{\partial A^0}{\partial y} - \frac{\partial A^2}{\partial t} = 0 \quad E_z = -\frac{\partial A^3}{\partial t}$$

$$B_x = \frac{\partial A^3}{\partial y} \quad B_y = -\frac{\partial A^3}{\partial x} \quad B_z = -\frac{\partial A^1}{\partial y} + \frac{\partial A^2}{\partial x} = 0, \quad (5)$$

where again we have assumed no z -dependence. It is clear that the emerging low-energy theory does not meet our expectations. The only unconstrained component of the gauge field is A^3 , precisely the one we would like to eliminate. However, it is instructive for the following to take one more step and derive the equation of motion of A^3 .

In order to do so, we study the behaviour of the gauge-invariant phase difference $\Delta\theta$ along the brane. We consider the path ABCD depicted in fig. 1. From eq. (3) with $\theta = 0$ we obtain

$$\begin{aligned} \Delta\theta_{AB} &= -q \int_A^B \vec{A} \cdot d\vec{l} \simeq \Delta\theta_{P_1 P_2} = -qdA^3(P) \\ \Delta\theta_{DC} &= -q \int_D^C \vec{A} \cdot d\vec{l} \simeq \Delta\theta_{Q_1 Q_2} = -qdA^3(Q) \\ \Delta\theta_{BC} &= \Delta\theta_{DA} = 0. \end{aligned} \quad (6)$$

Here $A^3(P)$, $A^3(Q)$ are the almost constant values of the gauge field inside the brane. We have neglected the contribution from the region of width $\lambda \sim (q\rho)^{-1}$, in which the gauge field falls off exponentially fast to zero. The superconducting currents in this region and, according to eq. (2), the gauge field are parallel to the surface of the brane to a good approximation. Along the line segments BC and DA the gauge field is zero. From the above equations we obtain

$$\frac{\partial(\Delta\theta)}{\partial x} = \frac{\Delta\theta_{DC} - \Delta\theta_{AB}}{\Delta x} \simeq -qd \frac{A_Q^3 - A_P^3}{\Delta x} = qdB_y. \quad (7)$$

By considering a path on the yz -plane and repeating the above steps we find

$$\frac{\partial(\Delta\theta)}{\partial y} = -qdB_x. \quad (8)$$

Finally, taking the time derivative of the first of eqs. (6) we find

$$\frac{\partial(\Delta\theta)}{\partial t} = qdE_z. \quad (9)$$

The above equations, eq. (4) and the Maxwell equations give

$$-qd \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} - \frac{\partial E_z}{\partial t} \right) + qdJ^3 = \partial^i \partial_i (\Delta\theta) + qdJ_{\max}^3 \sin(\Delta\theta) = 0, \quad (10)$$

where $i = 0, 1, 2$. We conclude that there is a mode on the brane that obeys the sine-Gordon equation. It is obvious from eqs. (3), (6) that this mode corresponds to the third component of the gauge field: $\Delta\theta = -qdA^3$. We consider weak fields ($\Delta\theta \ll 1$), for which we can approximate eq. (10) as

$$\left[\partial^i \partial_i + m^2 \right] \Delta\theta = 0, \quad (11)$$

with $m^2 = qdJ_{\max}^3$. In realistic Josephson junctions, eq. (11) implies the presence of a Meissner effect even in the non-superconducting material [13]. Applied electromagnetic fields decay over a distance $\sim (qdJ_{\max}^3)^{-1/2}$. This phenomenon has been observed experimentally. The decay length in the junction can be orders of magnitude larger than the decay length in the superconductor. Also solitonic configurations can appear, corresponding to solutions of eq. (10) [13].

From the point of view of obtaining an effective (2+1)-dimensional theory of electromagnetism, our attempt has failed. The 0,1,2-components of the gauge field are strongly constrained by eqs. (5), while the 3-component propagates freely but has a small mass. A possible remedy for this situation is provided by the suggestion of refs. [5, 6]. The material in the bulk must be a *dual* superconductor [15]. In other words, there must be a condensate of magnetic charge in the bulk.

It is believed that dual superconductivity is realized in the confining phase of gauge theories. The particular implementation of ref. [6] employs an $SU(2)$ gauge theory coupled to a scalar field in the adjoint representation (the Georgi-Glashow model) [16]. Inside the brane the $SU(2)$ symmetry is broken down to $U(1)$ through a non-zero expectation value of the scalar field. The low-energy theory is in the Coulomb phase and a massless photon should emerge³. In the bulk the scalar field has a zero expectation value and the theory is in the confining phase. All excitations are very massive, and this prevents the photon that is localized on the brane from entering the bulk. The connection with monopole condensation in the confining phase can be seen through 't Hooft's Abelian projection [18]. Any operator in the adjoint representation, such as the scalar field in this model, can be diagonalized through a gauge transformation that preserves an “electric” $U(1)$ gauge symmetry. This gauge transformation is singular at the points where the operator is zero. As a result, the zeros of various operator configurations can be considered as the world lines of “magnetic” monopoles. The latter are assumed to condense and confine “electric” charge. Support for the picture of monopole condensation in the confining phase has been obtained through lattice simulations [19].

In our discussion we shall use only the main elements of the above picture, since its details are not well understood. We consider electromagnetism in the presence of $U(1)$ magnetic charge. We assume that a magnetic condensate forms in the bulk, with the appearance of frictionless currents. In the absence of electric charge, we can use a phenomenological local Lagrangian description

$$\tilde{\mathcal{L}}_{tr} = \int d^4x \left\{ -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + [(\partial_\mu + igC_\mu) \psi]^* [(\partial^\mu + igC^\mu) \psi] - V(\psi) \right\}. \quad (12)$$

The dual gauge field C^μ is defined through the relation [20, 21]

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} F_{\lambda\sigma} = \partial^\mu C^\nu - \partial^\nu C^\mu, \quad (13)$$

and g is the magnetic charge. A frictionless magnetic current flows near the surface of the regions of non-zero expectation value for the magnetic condensate $\psi = \sigma \exp(i\chi)$. At tree level it is given by

$$\tilde{J}_{tr}^\mu = -2g\sigma^2 \partial^\mu \chi - 2g^2 \sigma^2 C^\mu. \quad (14)$$

A dual Meissner effect prevents the electric field from entering the regions with $\psi \neq 0$.

The system of fig. 1 can be viewed now as a dual Josephson junction with a tunnelling magnetic current \tilde{J}^3 flowing across the brane. It is clear from eq. (14) that the gauge-invariant definition of the phase difference $\Delta\chi$ between the two sides of the brane must involve the dual field C^μ . Repeating the arguments that led to eq. (4) we find

$$\tilde{J}^3(\Delta\chi) = \tilde{J}_{\max}^3 \sin(\Delta\chi). \quad (15)$$

³In fact, the electromagnetic field of the (2+1)-dimensional theory develops a small mass due to instanton effects and electric charge becomes confined with a linear potential [17]. However, this is related to the fact that the $U(1)$ symmetry is compact in the Georgi-Glashow model and has nothing to do with the phenomenon we discuss in this paper. Our approach is more general and our conclusions apply to non-compact $U(1)$ as well.

We emphasize at this point that the presence of a current is independent of the detailed form of the Lagrangian of the system. It is a consequence only of our assumption that a condensate exists on either side of the brane [14]. This is an important point, because a consistent Lagrangian description of a system with both electric and magnetic charges will be much more complicated than eq. (12) (and probably non-local) [21]. However, we believe that our arguments should be valid in the general case as well.

We turn next to the gauge field localized on the brane. We discuss its behaviour starting from the Maxwell equations, which we assume to be the correct equations of motion. It is sufficient for our discussion to consider a propagating electromagnetic field in the absence of electric charges or currents. The Maxwell equations read

$$\partial_\mu F^{\mu\nu} = 0, \quad (16)$$

$$\partial_\mu \tilde{F}^{\mu\nu} = \tilde{J}^\nu. \quad (17)$$

In the second equation we have included the tunnelling magnetic current \tilde{J}^3 . We must also take into account the constraints on the electromagnetic field arising from the presence of the dual superconducting phase in the bulk. The arguments we gave in the case of the standard Josephson junction can be repeated with the exchange of the role of electric and magnetic fields.

Let us assume for a moment that $\tilde{J}^\nu = 0$. Then eqs. (16), (17) can be solved in terms of the gauge field A^μ , as in our earlier discussion. The constraints from the presence of the dual superconducting phase in the bulk now give

$$\begin{aligned} E_x &= -\frac{\partial A^0}{\partial x} - \frac{\partial A^1}{\partial t} & E_y &= -\frac{\partial A^0}{\partial y} - \frac{\partial A^2}{\partial t} & E_z &= -\frac{\partial A^3}{\partial t} = 0 \\ B_x &= \frac{\partial A^3}{\partial y} = 0 & B_y &= -\frac{\partial A^3}{\partial x} = 0 & B_z &= -\frac{\partial A^1}{\partial y} + \frac{\partial A^2}{\partial x}, \end{aligned} \quad (18)$$

where again we have assumed no z -dependence. It is clear that A^3 is not a dynamical degree of freedom of the low-energy theory. The remaining components should belong to an effective (2+1)-dimensional gauge theory.

Taking into account the current \tilde{J}^ν in eq. (17) forbids a simple solution in terms of A^μ . One could try the ansatz [21]

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + \epsilon^{\mu\nu\lambda\sigma} G_{\lambda\sigma}. \quad (19)$$

Eq. (17) then gives

$$-\partial^\mu (G_{\mu\nu} - G_{\nu\mu}) = \tilde{J}_\nu. \quad (20)$$

The diagonal components of $G^{\mu\nu}$ do not contribute to eq. (19), and we take them to be zero. In our case the only non-zero component of the current is \tilde{J}^3 and we are looking for a solution with no z -dependence. This suggests $G^{ij} = 0$, with $i, j = 0, 1, 2$. Notice that we cannot take G^{ij} to be symmetric, because it would not contribute to eq. (19). This leaves us with G^{i3} , G^{3i} as possible non-zero components. An immediate consequence is that the expressions for B_x , B_y , E_z in eqs. (18) are not modified, and A^3 decouples from the low-energy theory.

The Maxwell equations can be solved easily in terms of the dual gauge field C^μ defined in eq. (13). The non-vanishing components of the electromagnetic field are

$$E_x = -\frac{\partial C^3}{\partial y} \quad E_y = \frac{\partial C^3}{\partial x} \quad B_z = -\frac{\partial C^3}{\partial t}, \quad (21)$$

where we have assumed no z -dependence. The reasoning that led to eq. (11) now gives

$$-gd\left(-\frac{\partial E_y}{\partial x} + \frac{\partial E_x}{\partial y} - \frac{\partial B_z}{\partial t}\right) + gd\tilde{J}^3 = \partial^i \partial_i (\Delta\chi) + gd\tilde{J}_{\max}^3 \sin(\Delta\chi) = 0. \quad (22)$$

For $\Delta\chi \ll 1$ we obtain

$$\left[\partial^i \partial_i + \tilde{m}^2\right] \Delta\chi = 0, \quad (23)$$

with $\tilde{m}^2 = gd\tilde{J}_{\max}^3$. The massive mode $\Delta\chi$ can be identified with the third component of the dual field: $\Delta\chi = -gdC^3$.

In summary, the following picture emerges: A (2+1)-dimensional theory appears on the brane. Its description in terms of the gauge field A^i , with $i = 0, 1, 2$, is complicated. However, in terms of the dual field C^3 , the physical quantities E_x , E_y , B_z are given by the simple expressions (21). The field C^3 is massive, with a mass \tilde{m} that can be very small in units of the typical scale of the theory in the bulk. As a result, the electromagnetic field E_x , E_y , B_z has a finite correlation length. This can be seen by simply taking y , x , t -derivatives of eq. (23) and remembering that $\Delta\chi = -gdC^3$. Indirect experimental support of this picture comes from the observation of a Meissner effect in standard Josephson junctions.

We expect the above conclusions to remain valid in a theory that includes electric charges on the brane. As we mentioned earlier, the Josephson effect is an immediate consequence of the presence of charged condensates and does not depend on the details of the underlying theory [14]. Therefore, the complications encountered in constructing a consistent theory of electric and magnetic charges are not expected to lead to significant modifications of our arguments. We mention that, in a consistent theory, electric and magnetic charges must satisfy Dirac's quantization condition [22]

$$qg = 2\pi n. \quad (24)$$

An interesting solution of the sine-Gordon equation (22) is given by [13]

$$\Delta\chi = 2\sin^{-1} \operatorname{sech}[\tilde{m}(x - x_0)]. \quad (25)$$

It corresponds to a defect localized near the line $x = x_0$. The electric field E_y is non-zero near x_0 and vanishes at distances $\gtrsim \tilde{m}^{-1}$ away from it. The phase $\Delta\chi$ changes by 2π as x goes from $-\infty$ to ∞ . We consider a defect at the center of the surface bounded by the path ABCD in fig. 1. By calculating the line integral for the dual gauge field along ABCD, it is easy to see that the defect carries unit electric flux $2\pi/g = q$, for $n = 1$ in eq. (24). The energy per unit length of the defect is $\sim \left(\tilde{J}_{\max}^3/g^3d\right)^{1/2}$ [13, 23]. Lines with larger electric flux correspond to solutions for which $\Delta\chi$ varies by multiples of 2π . It is conceivable that flux-carrying lines of this type may connect opposite electric charges on the brane.

There is a close similarity between the bulk and the thin region of width d that we are considering. On the brane the electromagnetic field is massive and defects exist that carry electric flux, similarly to the behaviour in the bulk. The most consistent interpretation of the emerging (2+1)-dimensional theory is that it displays confinement with a linear potential, but with a typical scale much smaller than the scale characterizing the theory in the bulk. This is in agreement with the experimental studies of standard Josephson junctions. In that case, the system behaves as if the superconducting properties extend over the whole structure including the barrier [23]. In a certain sense, this is caused by the electric condensate penetrating the barrier instead of ending abruptly at the surface. For the dual picture that we are considering,

we expect the magnetic condensate to behave in an analogous way. The implication is that dual superconducting behaviour, and therefore confinement, must be present inside the brane as well.

The relevance of the localization mechanism we discussed for the reduction of a (4+1)-dimensional theory to a (3+1)-dimensional one is not firmly established. The problem arises because the mechanism makes explicit use of four-dimensional electric-magnetic duality. It is not obvious how this duality can be generalized in higher dimensions. For this reason we do not attempt here a quantitative discussion of the realistic case. However, our basic argument, leading to the existence of the Josephson current, relies solely on the presence of a charged condensate on both sides of the brane. If the localization mechanism involves shielding of fields by frictionless currents in a charged condensate, this argument is very likely to remain valid. It seems reasonable to speculate that the penetration of the barrier by the condensate will result in behaviour similar to that in the bulk, with observable consequences such as an effective mass for the gauge field or channelling of flux lines into tubes.

The most obvious experimental signature for the scenario we considered would be the generated effective mass for the electromagnetic field. The presence of flux lines between electric charges is probably too difficult to detect because of the smallness of their energy per unit length. The scale of new physics, related to the width d of the brane and the confinement scale in the bulk, is expected to be at least of the order of TeV [7, 8]. The experimental upper bound on the photon mass is at the level of 10^{-16} eV (or possibly 10 orders of magnitude more stringent) [24]. This means that the effective current density $\tilde{J}_{\text{max}}^3 d$ across the brane must be $\lesssim 10^{-56}$ in units of the scale of new physics. As this current is a tunnelling one, such small values are not unnatural. The most exciting aspect of this scenario is that it has experimental low-energy implications for a non-gravitational sector. It demonstrates that the presence of extra dimensions can be unveiled much below the energies at which particles are released in the bulk.

We finish with a remark on the scenario with more than one extra dimensions. In the models of refs. [7, 8] the gauge fields are localized inside a vortex instead of a brane. The mechanism is analogous to the localization of magnetic fields in superconducting rings or macroscopic holes in superconducting materials. One important property of such geometries is that magnetic fields produced by a source do not disappear when this source is removed. Instead, they are maintained by frictionless currents at the surface of the superconductor. This is a more explicit manifestation of the presence of a condensate in the bulk. If the analogy with superconductivity persists for the reduction of a (5+1)-dimensional theory to a (3+1)-dimensional one, a clear experimental signature will be the time delay in the variation of electromagnetic fields with respect to the variation of the source that generates them.

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